More on Meta-Stable Brane Configuration by Quartic Superpotential for Fundamentals

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Abstract

For the case where the gauge theory superpotential has a quartic term as well as the mass term for quarks, the nonsupersymmetric meta-stable brane configuration was found recently. By adding the orientifold 6-planes and the extra fundamental flavors to this brane configuration, we describe the meta-stable nonsupersymmetric vacua of the gauge theory with antisymmetric flavor as well as fundamental flavors in type IIA string theory.

1 Introduction

It is known that the dynamical supersymmetry breaking in meta-stable vacua [1, 2] occurs in the standard $\mathcal{N} = 1$ SQCD with massive fundamental flavors. The extra mass term for quarks in the superpotential implies that some of the F-term equations cannot be satisfied and then the supersymmetry is broken. The corresponding meta-stable brane realizations of type IIA string theory have been found in [3, 4, 5]. Very recently Giveon and Kutasov [6, 7] have found the type IIA nonsupersymmetric meta-stable brane configuration where an additional quartic term for quarks in the superpotential is present. Geometrically, this extra deformation corresponds to the rotation of D6-branes along the (45)-(89) directions while keeping the other branes described in [3, 4, 5] unchanged. Classically there exist only supersymmetric ground states. By adding the orientifold 6-plane to this brane configuration [6], the meta-stable nonsupersymmetric vacua of the supersymmetric unitary gauge theory with symmetric flavor plus fundamental flavors is found [8].

Let us add an orientifold 6-plane and extra eight half D6-branes, located at the NS5'-brane, into the brane configuration of [6] together with an extra NS5-brane and the mirrors for both D4-branes and rotated D6-branes. According to the observation of [9, 10, 11], this "fork" brane configuration contains the NS5'-brane embedded in an O6-plane at $x^7 = 0$. This NS5'-brane divides the O6-plane into two separated regions corresponding to positive x^7 and negative x^7. Then RR charge of the O6-plane jumps from -4 to $+4$. Furthermore, eight semi-infinite D6-branes are present in the positive x^7 region. This is necessary for the vanishing of the six dimensional anomaly. Then the type IIA brane configuration consists of two NS5-branes, one NS5'-brane, D4-branes, rotated D6-branes, an O6-plane and eight half D6-branes. We'll see how the corresponding supersymmetric gauge theory, which is a standard $\mathcal{N} = 1$ SQCD with massive flavors together with the extra matters, occurs in the context of dynamical supersymmetry breaking in meta-stable vacua.

In this paper, we study $\mathcal{N} = 1$ $SU(N_c)$ gauge theory with an antisymmetric flavor A, a conjugate symmetric flavor $\widetilde{S}$, N_f fundamental flavors Q and $\widetilde{Q}$ and eight fundamental flavors $\hat{Q}$ in the context of dynamical supersymmetric breaking vacua. Now we deform this theory by adding both the mass term and the quartic term for quarks $Q, \widetilde{Q}$ in the fundamental representation of the gauge group [6]. Then we turn to the dual magnetic gauge theory [12]. The dual magnetic theory giving rise to the meta-stable vacua is described by $\mathcal{N} = 1$ $SU(2N_f - N_c + 4)$ gauge theory with dual matter contents. The difference between the brane configuration of [12] and the brane configuration of this paper is that the D6-branes are rotated in the (45)-(89) directions. By analyzing the magnetic superpotential, along the line

of [6, 7], we present the behaviors of gauge theory description and string theory description for the meta-stable vacua.

In section 2, the type IIA brane configuration corresponding to the electric theory based on the $\mathcal{N}=1$ $SU(N_c)$ gauge theory with above matter contents is given. In section 3, we construct the Seiberg dual magnetic theory which is $\mathcal{N}=1$ $SU(2N_f-N_c+4)$ gauge theory with corresponding dual matters. The rotation of D6-branes is encoded in the mass term for the meson field in the superpotential. In section 4, the nonsupersymmetric meta-stable minimum is found and the corresponding intersecting brane configuration of type IIA string theory is presented. In section 5, we comment on the future works.

2 The $\mathcal{N}=1$ supersymmetric electric brane configuration

The type IIA supersymmetric electric brane configuration [9, 10, 11, 12] corresponding to $\mathcal{N}=1$ $SU(N_c)$ gauge theory with an antisymmetric flavor A, a conjugate symmetric flavor $\widetilde{S}$, eight fundamental flavors $\hat{Q}$ and N_f fundamental flavors $Q,\widetilde{Q}$ [13] can be described as follows: one middle NS5'-brane(012389), two NS5-branes(012345) denoted by $NS5_L$-brane and $NS5_R$-brane respectively, N_c D4-branes(01236) between them, $2N_f$ D6-branes(0123789), an orientifold 6 plane(0123789) of positive RR charge, an orientifold 6 plane(0123789) of negative RR charge and eight half D6-branes. The transverse coordinates (x^4,x^5,x^6) transform as $(-x^4,-x^5,-x^6)$ under the orientifold 6-plane(O6-plane) action. Let us introduce two complex coordinates [14]

$$v\equiv x^4+ix^5, \qquad w\equiv x^8+ix^9.$$

Then the origin of the coordinates (x^6,v,w) is located at the intersection of x^6 coordinate and O6-plane. The left $NS5_L$-brane is located at the left hand side of O6-plane while the right $NS5_R$-brane is located at the right hand side of O6-plane. The N_c color D4-branes are suspended between $NS5_L$-brane and $NS5_R$-brane. Moreover the N_f D6-branes are located between the $NS5_L$-brane and the middle NS5'-brane and its mirrors N_f D6-branes are located between the middle NS5'-brane and the $NS5_R$-brane. The antisymmetric and conjugate symmetric flavors A and $\widetilde{S}$ are 4-4 strings stretching between D4-branes located at the left hand side of O6-plane and those at the right hand side of O6-plane, N_f fundamental flavors Q and $\widetilde{Q}$ are strings stretching between N_f D6-branes and N_c color D4-branes and eight fundamental flavors $\hat{Q}$ are strings stretching between eight half D6-branes which are on top of $O6^-$-plane and N_c color D4-branes.

Let us deform this theory which has vanishing superpotential by adding both the mass term and the quartic term for N_f fundamental quarks. The former can be achieved by "translating" the D6-branes along $\pm v$ direction leading to their coordinates $v = \pm v_{D6}$ [14] while the latter can be obtained by "rotating" the D6-branes [6] by an angle θ in (w, v)-plane. We denote them by $D6_\theta$-branes which are at angle θ with undeformed unrotated D6-branes(0123789). Then their mirrors N_f D6-branes are rotated by an angle $-\theta$ in (w, v)-plane according to O6-plane action and we denote them also by $D6_{-\theta}$-branes [1]. Then, in the electric gauge theory, the deformed superpotential is given by

$$\begin{aligned} W_{elec} &= \frac{\alpha}{2}\,\mathrm{tr}(Q\widetilde{Q})^2 - m\,\mathrm{tr}\,Q\widetilde{Q} - \frac{1}{2\mu}\left[(A\widetilde{S})^2 + Q\widetilde{S}A\widetilde{Q} + (Q\widetilde{Q})^2\right] + \hat{Q}\widetilde{S}\hat{Q}, \\ &\text{with} \qquad \alpha = \frac{\tan\theta}{\Lambda}, \qquad m = \frac{v_{D6}}{2\pi\ell_s^2} \end{aligned} \tag{2.1}$$

where Λ is related to the scales of the electric and magnetic theories and $\pm v_{D6}$ is the v coordinate of $D6_{\mp\theta}$-branes. Due to the last term, the flavor symmetry $SU(N_f+8)_L$ is broken to $SU(N_f)_L \times SO(8)_L$. Here the adjoint mass $\mu \equiv \tan(\frac{\pi}{2} - \omega)$ is related to a rotation angle ω of $NS5_{L,R}$-branes in (w, v)-plane. In the limit of $\mu \to \infty$(or no rotations of NS5-branes $\omega \to 0$), the terms of $\frac{1}{\mu}$ in (2.1) vanish.

Let us summarize the $\mathcal{N} = 1$ supersymmetric electric brane configuration with nonvanishing superpotential (2.1) in type IIA string theory as follows and draw it in Figure 1:

- Two NS5-branes in (012345) directions with $w = 0$
- One NS5'-brane in (012389) directions with $v = 0 = x^6$
- N_c color D4-branes in (01236) directions with $v = 0 = w$
- N_f $D6_{\pm\theta}$-branes in (01237) directions and two other directions in (v, w)-plane
- Eight half D6-branes in (0123789) directions with $x^6 = 0 = v$
- $O6^{\pm}$-planes in (0123789) directions with $x^6 = 0 = v$

By moving the $D6_{\pm\theta}$-branes from Figure 1 into the outside of $NS5_{L,R}$-branes, there exist N_f flavor D4-branes connecting $D6_{\pm\theta}$-branes and the $NS5_{L,R}$-branes, and the gauge singlet field N appears. At energies much below the mass of N, the two brane descriptions coincide with each other. One can think of this new brane configuration as integrating the field N in from Figure 1 and the superpotential of this electric theory contains the interaction term between N with electric quarks, quadratic term and linear term for N [6]. The classical supersymmetric vacua of this brane configuration are characterized by the parameter k where $k = 0, 1, \cdots, N_c$ and unbroken gauge symmetry in the k-th configuration is $SU(N_c - k)$. That

[1]Note that the convention for $D6_\theta$-branes in [12] was such that the angle between unrotated D6-branes and $D6_\theta$-branes was not θ but $(\frac{\pi}{2} - \theta)$.

is, the k D4-branes among N_f D4-branes (stretched between $NS5_R$-brane and $D6_{-\theta}$-branes) are reconnecting with those number of D4-branes stretched between the middle NS5'-brane and $NS5_R$-brane. Then those resulting k D4-branes are moving to $\pm v$ direction and the remaining $(N_c - k)$ D4-branes are stretching between the middle NS5'-brane and $NS5_R$-brane and $(N_f - k)$ D4-branes are stretched between the $NS5_R$-brane and $D6_{-\theta}$-branes(and their mirrors).

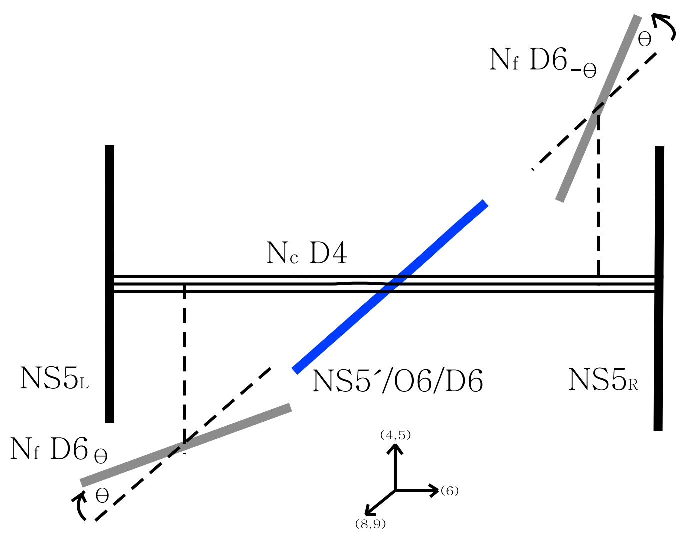

Figure 1: The $\mathcal{N} = 1$ supersymmetric electric brane configuration with deformed superpotential (2.1) for the $SU(N_c)$ gauge theory with an antisymmetric flavor A, a conjugate symmetric flavor $\widetilde{S}$, eight fundamentals $\hat{Q}$, and N_f fundamental massive flavors $Q, \widetilde{Q}$. The origin of the coordinates (x^6, v, w) is located at the intersection of x^6 and O6-plane. It is evident that the two deformations are characterized by both translation and rotation for D6-branes. As in [12], a combination of a middle NS5'-brane, $O6^+$-plane, $O6^-$-plane and eight half D6-branes is represented by $NS5'/O6/D6$.

3 The $\mathcal{N} = 1$ supersymmetric magnetic brane configuration

The magnetic theory is obtained by interchanging the $D6_{\pm\theta}$-branes and $NS5_{L,R}$-branes while the linking number is preserved. After one moves the left $D6_{\theta}$-branes to the right all the way(and their mirrors, right $D6_{-\theta}$-branes to the left) past the middle NS5'-brane and the right $NS5_R$-brane, the linking number counting [12] implies that one should add N_f D4-branes, corresponding to the meson $M(\equiv Q\widetilde{Q})$, to the left side of all the right N_f $D6_{\theta}$-branes(and their mirrors). Note that when a D6-brane crosses the middle NS5'-brane, due to

the parallelness of these, there was no creation of D4-branes. That is, when the $D6_{\pm\theta}$-branes approach the middle NS5'-brane, one should take $\theta = 0$ limit(making D6-branes to be parallel to the middle NS5'-brane) and then after they cross the middle NS5'-brane, they return to the original positions given by $D6_{\pm\theta}$-branes as follows:

$$D6_{\pm\theta} - \text{branes} \qquad \rightarrow \qquad D6 - \text{branes} \qquad \rightarrow \qquad D6_{\pm\theta} - \text{branes}. \tag{3.1}$$

Next, let us move the left $NS5_L$-brane to the right all the way past O6-plane (and its mirror, right $NS5_R$-brane to the left), and then the linking number counting [12] leads to the fact that the dual number of colors was $(2N_f - N_c + 4)$. There was a creation of D4-branes when the NS5-brane crosses an O6-plane because they are not parallel to each other. From this, the constant term 4 in the dual color above arises, compared with the case of [15]. Now we draw the magnetic brane configuration in Figure 2 where some of the flavor D4-branes are recombined with those of $(2N_f - N_c + 4)$ color D4-branes and those combined resulting D4-branes are moved into $\pm v$ direction. One takes k D4-branes from N_f flavor D4-branes and reconnect them to those from $(2N_f - N_c + 4)$ color D4-branes in Figure 2 such that the resulting branes are connecting from the $D6_\theta$-branes to the $NS5_L$-brane directly. Their coordinates between $D6_\theta$-branes and k D4-branes will be $v = -v_{D6}$ in order to minimize the energy. This Figure 2 also can be obtained from the magnetic brane configuration of [6] by adding O6-planes and eight half D6-branes with the right presence of mirrors under the O6-plane action.

Then the low energy dynamics is described by the dual magnetic theory with gauge group $SU(2N_f - N_c + 4)$ and this theory is higgsed down to $SU(2N_f - N_c + 4 - k)$ in the k-th vacuum by nonzero vacuum expectation value for dual quarks which is determined later. The matters are N_f flavors of fundamentals $q, \widetilde{q}$, an antisymmetric flavor a, a conjugate symmetric flavor $\widetilde{s}$, eight fundamentals $\hat{q}$, gauge singlet M which is magnetic dual of the electric meson field $Q\widetilde{Q}$ and other gauge singlet $\widetilde{M}$ that is $\hat{Q}\widetilde{Q}$. Then the superpotential including the interaction between the meson field M and dual matters with $\mu \rightarrow \infty (\omega \rightarrow 0)$ limit is described by

$$W_{mag} = \frac{1}{\Lambda} M q \widetilde{s} a \widetilde{q} + \frac{\alpha}{2} \operatorname{tr} M^2 - m \operatorname{tr} M + \hat{q} \widetilde{s} \hat{q} + \widetilde{M} \hat{q} \widetilde{q}, \qquad M \equiv Q\widetilde{Q}, \qquad \widetilde{M} \equiv \hat{Q}\widetilde{Q} \tag{3.2}$$

where the second and third terms originate from the two deformations (2.1), and the fourth term also comes from electric theory. The θ-dependent coefficient function, α, in front of quadratic term of the meson field also occurs in the geometric brane interpretation for the different supersymmetric gauge theory [16]. The $\alpha = 0$ limit reduces to the theory given by [12]. The parameters α and m are the same as before in (2.1) [2].

[2]For arbitrary rotation angles of $D6_{\pm\theta}$-branes and $NS5_{\pm\omega}$-branes, there exist also other meson fields

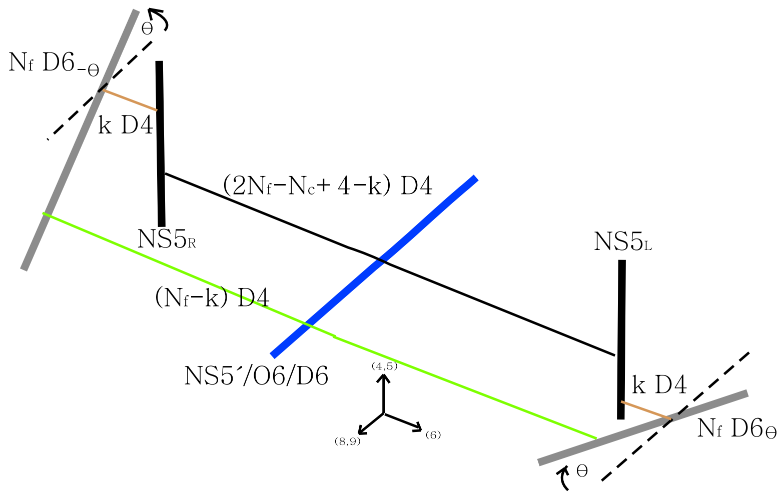

Figure 2: The $\mathcal{N}=1$ supersymmetric magnetic brane configuration for the $SU(2N_f - N_c + 4 - k)$ gauge theory with an antisymmetric flavor a, a conjugate symmetric flavor $\widetilde{s}$, N_f fundamental flavors $q, \widetilde{q}$ and eight fundamentals $\hat{q}$. The N_f flavor D4-branes connecting between $NS5_L$-brane and $D6_\theta$-branes are related to the dual gauge singlet M and are splitting into $(N_f - k)$ and k D4-branes. The location of intersection between $D6_\theta$-branes and (N_f-k) D4-branes is given by $(v,w) = (0, v_{D6}\cot\theta)$ while the one between $D6_\theta$-branes and k D4-branes is given by $(v,w) = (-v_{D6}, 0)$.

From the magnetic superpotential (3.2), the supersymmetric vacua are obtained and the F-term equations are given as follows:

$$\begin{aligned}
\widetilde{s}a\widetilde{q}M &= 0, \qquad a\widetilde{q}Mq + \hat{q}\hat{q} = 0, \\
\widetilde{q}Mq\widetilde{s} &= 0, \qquad Mq\widetilde{s}a + \widetilde{M}\hat{q} = 0, \\
\widetilde{s}\hat{q} + \widetilde{q}\widetilde{M} &= 0, \qquad \hat{q}\widetilde{q} = 0, \\
\frac{1}{\Lambda}q\widetilde{s}a\widetilde{q} &= m - \alpha M.
\end{aligned} \tag{3.3}$$

By multiplying M into the last equation with $\hat{q} = 0 = \widetilde{M}$, the matrix equation is satisfied $mM = \alpha M^2$. Because the eigenvalues are either 0 or $\frac{m}{\alpha}$, one can take $N_f \times N_f$ matrix with

containing A or $\widetilde{S}$:$M_1 \equiv Q\widetilde{S}A\widetilde{Q}, P \equiv Q\widetilde{S}Q$ and $\widetilde{P} \equiv \widetilde{Q}A\widetilde{Q}$. They couple to the dual quarks and other flavors in the superpotential via $M_1 q\widetilde{q} + Pq\widetilde{s}q + \widetilde{P}\widetilde{q}a\widetilde{q}$. As emphasized in [12], the geometric constraint (3.1) at the intersection between D6-branes and the middle NS5'-brane removes the presence of these gauge singlets, M_1, P and $\widetilde{P}$. That is, when $D6_{\pm\theta}$-branes are crossing the middle NS5'-brane, they do not produce any D4-branes because they are parallel at the origin $x^6 = 0$. This implies there is no M_1 term in the magnetic superpotential. In general, P and $\widetilde{P}$-terms arise when $D6_\theta$-branes intersect with its mirrors $D6_{-\theta}$-branes around the origin $x^6 = 0$. But they are also parallel to each other due to (3.1). This leads to the fact that there are no P or $\widetilde{P}$-terms in the magnetic superpotential. Therefore, we are left with (3.2).

k's eigenvalues 0 and $(N_f - k)$'s eigenvalues $\frac{m}{\alpha}$:

$$M = \begin{pmatrix} 0 & 0 \\ 0 & \frac{m}{\alpha}\mathbf{1}_{N_f-k} \end{pmatrix} \tag{3.4}$$

where $k = 1, 2, \cdots, N_f$ and $\mathbf{1}_{N_f-k}$ is the $(N_f - k) \times (N_f - k)$ identity matrix [7]. The expectation value of M is represented by the fundamental string between the flavor brane displaced by the w direction and the color brane from Figure 2. The k of the N_f flavor D4-branes are connected with k of $(2N_f - N_c + 4)$ color D4-branes and the resulting D4-branes stretch from the $D6_\theta$-branes to the $NS5_L$-brane and the coordinate of an intersection point between the k D4-branes and the $NS5_L$-brane is given by $(v, w) = (-v_{D6}, 0)$. This corresponds to exactly the k's eigenvalues 0 of M above. Now the remaining $(N_f - k)$ flavor D4-branes between the $D6_\theta$-branes and the NS5'-brane are related to the corresponding eigenvalues of M given by $\frac{m}{\alpha}\mathbf{1}_{N_f-k}$. The coordinate of an intersection point between the $(N_f - k)$ D4-branes and the NS5'-brane is given by $(v, w) = (0, v_{D6} \cot\theta)$. Note that using the expressions for α and m from (2.1), one obtains $\frac{m}{\alpha} = \Lambda \frac{v_{D6}\cot\theta}{2\pi\ell_s^2}$ which must be $\Lambda\frac{w}{2\pi\ell_s^2}$. Then $w = v_{D6}\cot\theta$.

Substituting (3.4) into the last equation of (3.3) leads to

$$q\widetilde{s}a\widetilde{q} = \begin{pmatrix} m\Lambda\mathbf{1}_k & 0 \\ 0 & 0 \end{pmatrix}. \tag{3.5}$$

Since the rank of the left hand side is at most $2N_f - N_c + 4$, one must have more stringent bound $k \leq (2N_f - N_c + 4)$. In the k-th vacuum the gauge symmetry is broken to $SU(2N_f - N_c + 4 - k)$ and the supersymmetric vacuum drawn in Figure 2 with $k = 0$ has $q\widetilde{s} = a\widetilde{q} = 0$ and the gauge group $SU(2N_f - N_c + 4)$ is unbroken. The expectation value of M in this case is given by $M = \frac{m}{\alpha}\mathbf{1}_{N_f} = m\Lambda\cot\theta\mathbf{1}_{N_f}$ explicitly. By moving the D6-branes into the place between the middle NS5'-brane and the $NS5_{L,R}$-branes, one obtains other brane configuration. There exist $(N_f - k)$ flavor D4-branes connecting $D6_{\pm\theta}$-branes and the $NS5_{L,R}$-branes after this movement.

Another deformation arises when we rotate the $NS5_{L,R}$-branes by an angle $\pm\theta'$ in the (v, w)-plane. This rotation provides an adjoint field of $SU(2N_f - N_c + 4)$ and couples to the magnetic dual matters. Integrating this adjoint field out leads to the fact that there exists a further contribution to the quartic superpotential for q and $\widetilde{q}$. In particular, when the rotated $NS5_{\pm\theta'}$-branes are parallel to rotated $D6_{\pm\theta}$-branes, the coupling in front of M^2 in the magnetic superpotential vanishes.

4 Nonsupersymmetric meta-stable brane configuration

The theory has many nonsupersymmetric meta-stable ground states besides the supersymmetric ones we discussed in previous section. For the IR free region [12], the magnetic theory is the effective low energy description of the asymptotically free electric gauge theory. When we rescale the meson field as $M = h\Lambda\Phi$, then the Kahler potential for Φ is canonical and the magnetic quarks are canonical near the origin of field space. The higher order corrections of Kahler potential are negligible when the expectation values of the fields $q, \widetilde{q}, a, \widetilde{s}$ and Φ are smaller than the scale of magnetic theory. Then the magnetic superpotential can be written in terms of Φ(or M)

$$W_{mag} = h\Phi q\widetilde{s}a\widetilde{q} + \frac{h^2\mu_\phi}{2}\,\mathrm{tr}\,\Phi^2 - h\mu^2\,\mathrm{tr}\,\Phi + \hat{q}\widetilde{s}\hat{q} + \widetilde{M}\hat{q}\widetilde{q}.$$

From this, one can read off the following quantities

$$\mu^2 = m\Lambda, \qquad \mu_\phi = \alpha\Lambda^2, \qquad M = h\Lambda\Phi.$$

The classical supersymmetric vacua given by (3.4) and (3.5) can be described similarly and one decomposes the $(N_f - k) \times (N_f - k)$ block at the lower right corner of $h\Phi$ and $q\widetilde{s}a\widetilde{q}$ into blocks of size n and $(N_f - k - n)$ as follows:

$$h\Phi = \begin{pmatrix} 0 & 0 & 0 \\ 0 & h\Phi_n & 0 \\ 0 & 0 & \frac{\mu^2}{\mu_\phi}\mathbf{1}_{N_f-k-n} \end{pmatrix}, \qquad q\widetilde{s}a\widetilde{q} = \begin{pmatrix} \mu^2\mathbf{1}_k & 0 & 0 \\ 0 & \varphi\widetilde{\beta}\gamma\widetilde{\varphi} & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Here φ and $\widetilde{\varphi}$ are $n \times (2N_f - N_c + 4 - k)$ dimensional matrices and correspond to n flavors of fundamentals of the gauge group $SU(2N_f - N_c + 4 - k)$ which is unbroken by the nonzero expectation value of $q\widetilde{s}$ and $a\widetilde{q}$. In Figure 3, they correspond to fundamental strings connecting the n flavor D4-branes and $(2N_f - N_c + 4 - k)$ color D4-branes. This Figure 3 can be also obtained from the meta-stable brane configuration of [6] by adding O6-planes and eight half D6-branes with appropriate mirrors under the O6-plane action. The antisymmetric and conjugate symmetric flavors γ and $\widetilde{\beta}$ are 4-4 strings stretching between $(2N_f - N_c + 4 - k)$ D4-branes located at the left hand side of O6-plane and those at the right hand side of O6-plane in Figure 3. Both Φ_n and $\varphi\widetilde{\beta}\beta\widetilde{\varphi}$ are $n \times n$ matrices. The supersymmetric ground state corresponds to the vacuum expectation vaules given by $h\Phi_n = \frac{\mu^2}{\mu_\phi}\mathbf{1}_n, \varphi\widetilde{\beta} = 0 = \gamma\widetilde{\varphi}$.

Now the full one loop potential for $\Phi_n, \hat{\varphi} \equiv \varphi\widetilde{\beta}, \hat{\widetilde{\varphi}} \equiv \gamma\widetilde{\varphi}$ [12] takes the form

$$\frac{V}{|h|^2} = |\Phi_n\hat{\varphi}|^2 + |\Phi_n\hat{\widetilde{\varphi}}|^2 + |\hat{\varphi}\hat{\widetilde{\varphi}} - \mu^2\mathbf{1}_n + h\mu_\phi\Phi_n|^2 + b|h\mu|^2\,\mathrm{tr}\,\Phi_n^\dagger\Phi_n,$$

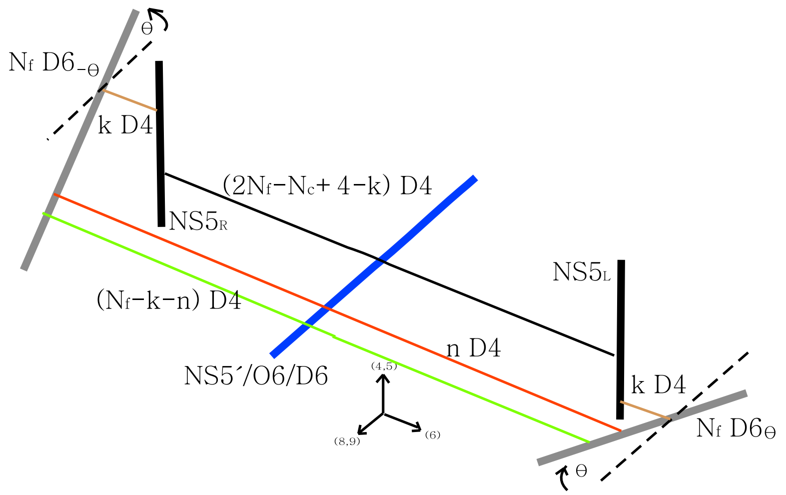

Figure 3: The nonsupersymmetric minimal energy brane configuration for the $SU(2N_f - N_c + 4 - k)$ gauge theory with an antisymmetric flavor a, a conjugate symmetric flavor $\widetilde{s}$, N_f fundamental flavors $q, \widetilde{q}$ and eight fundamentals $\hat{q}$. This brane configuration is obtained by moving n flavor D4-branes, from $(N_f - k)$ flavor D4-branes stretched between the NS5'-brane and the $D6_\theta$-branes in Figure 2(and their mirrors).

where b is given by $b = \frac{(\ln 4 - 1)}{8\pi^2}(2N_f - N_c + 4)$. Differentiating this potential with respect to Φ_n and putting $\hat{\varphi} = 0 = \hat{\tilde{\varphi}}$, one obtains

$$h\Phi_n = \frac{\mu^2 \mu_\phi^*}{|\mu_\phi|^2 + b|\mu|^2} \mathbf{1}_n \simeq \frac{\mu^2 \mu_\phi^*}{b|\mu|^2} \mathbf{1}_n \qquad \text{or} \qquad M_n \simeq \frac{\alpha \Lambda^3}{(2N_f - N_c + 4)} \mathbf{1}_n \tag{4.1}$$

for real μ and we assume here that $\mu_\phi << \mu << \Lambda_m$. The vacuum energy V is given by $V \simeq n|h\mu^2|^2$. Expanding around this solution, one obtains the eigenvalues for mass matrix for $\hat{\varphi}$ and $\hat{\tilde{\varphi}}$ will be

$$m_\pm^2 = \frac{|\mu|^4}{(|\mu_\phi|^2 + b|\mu|^2)^2} \left[|\mu_\phi|^2 \pm b|h|^2 \left(|\mu_\phi|^2 + b|\mu|^2 \right) \right] \simeq \frac{1}{b^2} \left(|\mu_\phi|^2 \pm |bh\mu|^2 \right).$$

Then for $|\frac{\mu_\phi}{\mu}|^2 > \frac{|bh|^2}{1 - b|h|^2} \simeq |bh|^2$ in order to avoid tachyons the vacuum (4.1) is locally stable.

One can move n D4-branes, from $(N_f - k)$ D4-branes stretched between the NS5'-brane and the $D6_\theta$-branes at $w = v_{D6} \cot\theta$, to the local minimum of the potential and the end points of these n D4-branes are at a nonzero w as in Figure 3. The remaining $(N_f - k - n)$ flavor D4-branes between the $D6_\theta$-branes and the NS5'-brane are related to the corresponding eigenvaules of $h\Phi$, i.e., $\frac{\mu^2}{\mu_\phi} \mathbf{1}_{N_f - k - n}$. The coordinate of an intersection point between the $(N_f - k - n)$ D4-branes and the NS5'-brane is given by $(v, w) = (0, v_{D6} \cot\theta)$. Finally, the remnant n "curved" flavor D4-branes between the $D6_\theta$-branes and the NS5'-brane are related to the corresponding eigenvaules of $h\Phi_n$ by (4.1). Note that since $\frac{\mu^2 \mu_\phi^*}{b|\mu|^2} << \frac{\mu^2}{\mu_\phi}$, the n D4-branes are nearer to the $w = 0$ located at the $NS5_L$-brane.

As explained in [6], this local stable vacuum decays to the supersymmetric ground states. The end points of n "curved" flavor D4-branes on the NS5'-brane approach those of the $(2N_f - N_c + 4 - k)$ color D4-branes and two types of branes reconnect each other. For $n \leq (2N_f - N_c + 4 - k)$, the final brane configuration is nothing but the supersymmetric vacuum of Figure 2 with the replacement $k \to (k+n)$. When $n > (2N_f - N_c + 4 - k)$, then the remnant $[n - (2N_f - N_c + 4 - k)]$ flavor D4-branes remain. On the other hand, the n D4-branes can move to larger w and return to the Figure 2. Also some of the D4-branes approach the intersection point between $D6_\theta$-branes and the NS5'-brane while the remaining D4-branes move to the one between $D6_\theta$-branes and the $NS5_L$-brane.

When $D6_{\pm\theta}$-branes are moved to the place between the NS5'-brane and the $NS5_{L,R}$-branes, one gets the Figure 4 where the previous $(N_f - k)$ D6-branes in Figure 3 that were not connected to the $NS5_{L,R}$-branes, through the flavor D4-branes, are now connecting to the $NS5_{L,R}$-branes by the same number of D4-branes while the previous k D6-branes in Figure 3 that were connected to the $NS5_{L,R}$-branes, through the flavor D4-branes, are now not connecting to the $NS5_{L,R}$-branes. The former corresponds to a creation of D4-branes and the latter corresponds to an annihilation of D4-branes due to the Hanany-Witten transition [17, 14].

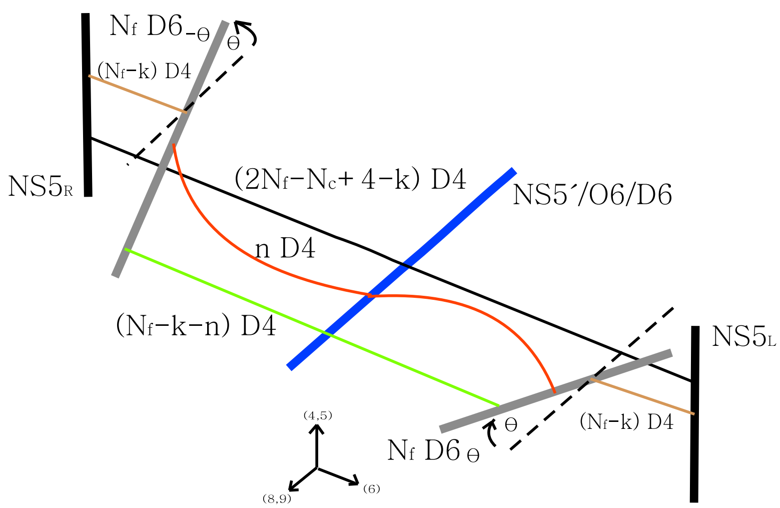

Figure 4: The nonsupersymmetric minimal energy brane configuration for the $SU(2N_f - N_c + 4 - k)$ gauge theory with an antisymmetric flavor a, a conjugate symmetric flavor $\widetilde{s}$, N_f fundamental flavors $q, \widetilde{q}$ and eight fundamentals $\hat{q}$. This brane configuration can be obtained by moving N_f $D6_{\pm\theta}$-branes into the place between the $NS5_{L,R}$-branes and the middle NS5'-brane in Figure 3(and their mirrors). Note that there exists a creation of $(N_f - k)$ D4-branes connecting the $D6_\theta$-branes and the $NS5_L$-brane(and their mirrors).

In the remaining part, we focus on the gravitational potential of the $NS5_L$-brane. Let us

remind that the branes are placed as follows:

$$
\begin{array}{rcl}
D6_\theta - \text{branes}(01237vw) & : & v = -v_{D6} + w\tan\theta, \\
NS5' - \text{brane}(012389) & : & v = 0, \\
NS5_L - \text{brane}(012345) & : & w = 0
\end{array}
$$

where we assume that $D6_\theta$-branes are located at the nonzero

$$y \equiv x^6$$

and the $NS5'$-brane is located at $y = 0$(the origin). The dependence of the distance between the $D6_\theta$-branes and $NS5'$ brane along the $NS5_L$-brane on w can be represented by

$$\Delta x = | - v_{D6} + w\tan\theta|. \tag{4.2}$$

Then the partial differentiation of Δx with respect to w leads to an extra contribution for the computation of $\partial_w \theta_{i,w}$ where we define $\theta_{i,w}$ as

$$\theta_{1,w} \equiv \cos^{-1}\left(\frac{y_m}{|w|}\right), \qquad \theta_{2,w} \equiv \cos^{-1}\left(\frac{y_m}{\sqrt{y^2+|w|^2}}\right) \tag{4.3}$$

with y_m that is smallest value of y along the D4-brane. Then the energy density of the D4-brane was computed in [18, 19] and is given by

$$E(w) = \frac{\tau_4}{2\ell_s}\sqrt{1+\frac{\ell_s^2}{y_m^2}}\left[|w|^2\sin 2\theta_{1,w} + (y^2+|w|^2)\sin 2\theta_{2,w}\right] \tag{4.4}$$

corresponding to (3.7) of [6] where τ_4 is a tension of D4-brane in flat spacetime.

It is straightforward to compute the differentiation of $\left(\frac{\ell_s E(w)}{\tau_4}\right)^2$ with respect to w and it leads to

$$
\begin{aligned}
\partial_w\left(\frac{\ell_s E(w)}{\tau_4}\right)^2 &= \ell_s^2 \bar{w}\left(\sin^2\theta_{1,w} + \sin^2\theta_{2,w} + \left[\frac{\sqrt{y^2+|w|^2}}{|w|} + \frac{|w|}{\sqrt{y^2+|w|^2}}\right]\sin\theta_{1,w}\sin\theta_{2,w}\right) \\
&+ \left[|w|^2\sin 2\theta_{1,w} + \left(y^2+|w|^2\right)\sin 2\theta_{2,w}\right] \times \Big[\left(\ell_s^2 + |w|^2\cos 2\theta_{1,w}\right)\partial_w\theta_{1,w} \\
&+ \left(\ell_s^2 + \left(y^2+|w|^2\right)\cos 2\theta_{2,w}\right)\partial_w\theta_{2,w} + \frac{\bar{w}}{2}\left(\sin 2\theta_{1,w} + \sin 2\theta_{2,w}\right)\Big].
\end{aligned} \tag{4.5}
$$

In order to simplify this, one uses the partial differentiation of Δx (4.2) with respect to w which is equal to $\frac{1}{2}\tan\theta\frac{\bar{w}\tan\bar{\theta} - v_{\bar{D}6}}{|w\tan\theta - v_{D6}|}$. On the other hand, Δx was known in [18] and it is

$$\Delta x = \frac{1}{2\ell_s}\left[|w|^2\sin 2\theta_{1,w} + (y^2+|w|^2)\sin 2\theta_{2,w}\right] + \ell_s\left(\theta_{1,w} + \theta_{2,w}\right). \tag{4.6}$$

After differentiating this (4.6) with respect to w and equating it to the previous expression obtained from (4.2), one arrives at

$$\left(\ell_s^2+|w|^2\cos 2\theta_{1,w}\right)\partial_w\theta_{1,w}+\left[\ell_s^2+\left(y^2+|w|^2\right)\cos 2\theta_{2,w}\right]\partial_w\theta_{2,w}$$
$$+\frac{\bar{w}}{2}\left(\sin 2\theta_{1,w}+\sin 2\theta_{2,w}\right)=\ell_s\frac{1}{2}\tan\theta\frac{\bar{w}\bar{\tan}\theta-\bar{v_{D6}}}{|w\tan\theta-v_{D6}|} \quad (4.7)$$

corresponding to (3.21) of [6]. Now by putting this relation (4.7) into the second and third lines of (4.5), one gets

$$\partial_w\left(\frac{\ell_s E(w)}{\tau_4}\right)^2 = \ell_s^2\bar{w}\left(\sin^2\theta_{1,w}+\sin^2\theta_{2,w}+\left[\frac{\sqrt{y^2+|w|^2}}{|w|}+\frac{|w|}{\sqrt{y^2+|w|^2}}\right]\sin\theta_{1,w}\sin\theta_{2,w}\right)$$
$$+ \left[|w|^2\sin 2\theta_{1,w}+\left(y^2+|w|^2\right)\sin 2\theta_{2,w}\right]\frac{1}{2}\ell_s\tan\theta\frac{\bar{w}\bar{\tan}\theta-\bar{v_{D6}}}{|w\tan\theta-v_{D6}|}. \quad (4.8)$$

It is easy to see that at $w=v_{D6}\cot\theta$ which is an intersection point between $D6_\theta$-branes and $NS5'$-brane, Δx vanishes through (4.2) and this also implies that $\theta_{i,w}$ vanishes from (4.6). Furthermore, the energy $E(w)$ is zero from (4.4). This corresponds to the global minimal energy. For the parallel $D6$-branes and NS5'-brane(i.e., $\tan\theta=0$), then the only stationary point is $w=0$ [12]. If $w\neq 0$, then $\sin\theta_{1,w}=0=\sin\theta_{2,w}$ from the first term of (4.8) but these are not physical solutions.

For real and positive parameters v_{D6}, w and $\tan\theta$, we are looking for the solution with $v_{D6}>w\tan\theta$ and setting the right hand side of (4.8) to zero, finally one gets with (4.3)

$$\frac{\sin^2\theta_{1,w}+\sin^2\theta_{2,w}+\left(\frac{\sqrt{y^2+w^2}}{w}+\frac{w}{\sqrt{y^2+w^2}}\right)\sin\theta_{1,w}\sin\theta_{2,w}}{w^2\sin 2\theta_{1,w}+(y^2+w^2)\sin 2\theta_{2,w}}=\frac{\tan\theta}{2\ell_s w}. \quad (4.9)$$

Therefore, the brane configuration of Figure 4 has a local minimum where the end of D4-brane are located at w given by (4.9). When the y goes to zero($\theta_{1,w}=\theta_{2,w}\equiv\theta_w$), one can approximate (4.9) and one gets

$$\tan\theta_w\simeq\frac{w\tan\theta}{2\ell_s}.$$

The gauge theory result is valid only when θ and $\frac{v_{D6}}{\ell_s}$ are much smaller than g_s while the classical brane construction with (4.9) is valid for any angle and the length parameters are of order ℓ_s or larger.

5 Conclusions and outlook

In this paper, by adding the orientifold 6-planes and the extra fundamental flavors to the brane configuration [6], we have described the meta-stable nonsupersymmetric vacua of the

gauge theory with antisymmetric flavor as well as fundamental flavors, through the Figures 3 and 4, from type IIA string theory.

It would be interesting to deform the theories given in [20, 19, 21, 22, 23] where one of the gauge group factor has the same matter contents as the one of the present paper and see how the meta-stable ground states appear. Along the lines of [18, 19, 24, 21, 23], when the $D6_\theta$-branes are replaced by a single $NS5_\theta$-brane, it would be interesting to see how the present deformation arises in these theories. It is also possible to deform the symplectic or orthogonal gauge group theory with massive flavors [25] by adding an orientifold 4-plane. Similar application to the product gauge group case [26, 27, 28] is also possible to study. It is an open problem to see how the type IIB description [29] is related to the present work. To construct a direct gauge mediation [30, 31] for the present work is also possible open problem.

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